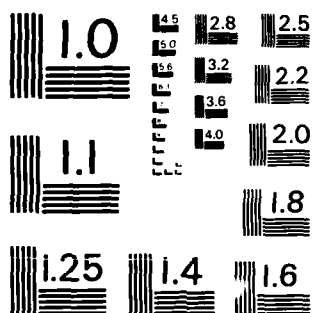


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SOME RECOMMENDATIONS FOR IMPROVEMENTS IN THE THEORY AND PRACTICE
OF DOD INCENTIVE CONTRACTING

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24 August 1984

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) Fundamental theoretical deficiencies are shown and revision is recommended in the DoD's newly proposed procedure for adjusting shared savings to encourage cost reducing capital investment by contractors. It is also shown that inconsistencies between conceptually correct economic figures and their counterparts computed under government accounting standards cause underallowance of intended contractor profit incentive. Also, though standard models under uncertainty allow the contractor to choose his sharing fraction for cost underruns and overruns so as to maximize his expected utility or, equivalently, his risk-adjusted value, it is shown that this approach does not, in general, yield Pareto-optimality and hence is not in the best interest of either the contractor or the government. Contrary to a claim in the literature, it is further shown that sharing may well be an incentive for cost reduction, but care must be used to assure Pareto-optimality in the final solution. Recommendations are made for follow-on research including joint consideration of time streams and uncertainty, multiattribute analysis,					
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19. and other extensions in the theory. ↑

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I. INTRODUCTION

Incentive contracting, i.e. the correlation of contractor fees with contractor performance, has been studied and used in USAF, DOD, and other U.S. Government acquisition programs for many years. Indeed Scherer's^{1,2} seminal theoretical treatment of its application to acquisition of advanced weapon systems appeared 20 years ago. However, heightened interest in the topic has recently been caused by establishment of the DOD's Industrial Modernization Incentives Program and circulation of a draft of DOD Instruction 5000.XX:DOD Guide for Improving Productivity in Defense Contracting.³ As a result, for the summer research described below, given the present author's prior background in the theory of managerial and engineering economics, he has been asked to begin a thorough examination and critique of the theory of incentive contracting, particularly as it applies to military acquisition programs, and to make recommendations for revisions and extensions in the theory and improvements in its application.

II. OBJECTIVES

The following specific objectives were pursued in this initial phase of the investigation:

- 1) The DOD Guide³ proposes a specific procedure for encouraging contractors to make further capital investment for cost reduction by having the government revise the manner in which it shares resulting savings with the contractors. Study was made of the fundamental conflict between this proposed procedure and standard engineering economic theory.
- 2) In application of incentive contracting theory there can easily be a discrepancy between numerical figures procured under required accounting standards vs. the figures which are logically correct from a proper economic point-of-view. An initial study was made of how serious this conflict might be.
- 3) For the handling of uncertainty, previous authors have developed simple models which allow the contractor to choose the degree to which he wishes to share savings

and costs with the government. But, in the present analysis, these models were put into a larger context in which the risk attitudes of both the contractor and the government were considered. Conflict was explored between previously-recommended solutions and those which are Pareto-optimal from an overall point of view.

- 4) Finally, in the light of the above studies and further examination of the literature, recommendations were made proposing follow-on research which shows significant promise of yielding additional insights for useful extension of incentive contracting theory and its practical application.

III. DOD'S SHARED SAVINGS PROCEDURE IS FUNDAMENTALLY UNSOUND

For simplicity, assume certainty and a perfect capital market in which money may be borrowed or lent at a period interest rate, i , compounded periodically. In this setting, other things being equal, it is optimal for a capital investment project to be undertaken if and only if it has prospective cash flows for which the net present value is positive. See, e.g., Bernhard.⁴ But, as illustrated in the DOD Guide³, when a contractor has a prospective cost-reducing project which satisfies this requirement, a peculiarity of the "shared savings" provisions in typical government contracts is that the portion of cash flows allowed to the contractor has only a negative net present value, and hence he has no incentive to undertake the project or even report its existence.

Though the obvious solution would be to revise the sharing so that both the government's and the contractor's portions had positive net present values, this, regrettably, is not what the DOD Guide recommends. Rather, presuming that the contractor has a minimum acceptable internal rate of return, $i^* > i$, it is proposed that the shared savings be revised such that the contractor's cash flows for the project have that internal rate. But it is easy to

show, as follows, that this DOD procedure is fundamentally unsound.

With all monetary figures in millions of dollars, let:

$A_t \equiv$ Net time t cash flow from the project to the contractor.

$t = 0, 1, \dots, T.$

$NPV(i) \equiv$ Net time 0 present value of these flows to the contractor.

$$= A_0 + A_1 (1+i)^{-1} + A_2 (1+i)^{-2} + \dots + A_T (1+i)^{-T}.$$

Then, to satisfy the DOD requirement, the A_t 's must be set so

that $NPV(i^*)=0$.

E.g., consider a numerical illustration in which $i=0.10$, $i^*=0.20$, and $T=2$, and suppose that, under the initial shared savings provision, $A_0 = -1$, $A_1 = 0.19$, and $A_2 = -0.05$. Here $NPV(i) < 0$ for all $i \geq 0$, and no real internal rate exists; hence the contractor would clearly find this cash flow pattern unattractive. Then, following the DOD procedure, if A_0 is held fixed, the pairs, (A_1, A_2) , which make $i^*=0.20$ are any which satisfy (2).

$$A_2 = -1.2A_1 + 1.44 \quad (2)$$

E.g., the pattern, $(A_1=2.40, A_2=-1.44)$, qualifies. But note that even though $NPV(.20)=0$ here, $NPV(i) < 0$ for all other values of $i \geq 0$.

Thus, since there is no value of i which makes $NPV(i) > 0$, the contractor, contrary to what the DOD analysis implies, could not possibly find this cash flow pattern attractive either.

To help in understanding what is happening here, let:

$V_t \equiv$ Contractor's unrecovered investment in the project at time t .

$$= V_{t-1}(1+i^*) - A_t \text{ for } t=1, 2, \dots, T-1. \quad (3)$$

Then if $V_0 = -A_0$, it is easy to verify that $V_T = 0$. See, e.g., Bernhard.⁴ So, for the numerical situation above, $V_0 = 1 > 0$, but $V_1 = -1.2 < 0$. The former implies that the contractor would be, in effect, a lender to the project from $t=0$ till $t=1$, but the latter implies he would be a borrower from it from $t=1$ till $t=2$. In the former case he would indeed want $i^* > i$, but in the latter he would want $i^* < i$. Since he obviously cannot have both, the i^* index is simply not appropriate for use here or in any situation where V_t changes sign over time. For a fuller discussion of this point, see Bernhard⁴. Incidentally, for the 9-period

example presented in the DOD Guide³, $V_t > 0$ for $t=0$ through $t=4$, but $V_t < 0$ for $t=5$ through $t=8$; thus the i^* index is not appropriate for use there either.

Furthermore, even if consideration is limited to cash flow patterns for which $V_t \geq 0$ throughout, and hence for which it is indeed attractive for the contractor to have $i^* > i$, it is important to recognize that, again contrary to what the DOD analysis implies, not all cash flow patterns with the same i^* will be equally attractive to him; i.e., not all will have the same $NPV(i)$. E.g., two further patterns satisfying (2) above, and hence having the common $i^*=0.20$, are $(A_1=1.2, A_2=0)$, and $(A_1=0, A_2=1.44)$. Though each has $V_t \geq 0$ throughout, for the former $NPV(.10)=0.09$, while for the latter $NPV(.10)=0.19$. Hence, in monetary value, the contractor will find the latter more than twice as attractive as the former.

Finally, again with $V_t \geq 0$ throughout, note that a contractor may actually prefer a cash flow pattern with a lower i^* over one with a higher i^* . E.g., with A_0 still fixed, the pattern, $(A_1=0, A_2=1.42)$ has only $i^*=0.19$, but $NPV(.10)=0.17$. This is clearly preferable to the pattern, $(A_1=1.2, A_2=0)$, with its higher $i^*=0.20$, but its lower $NPV(.10)=0.09$. Thus the internal rate is simply fundamentally unsound as an index of goodness of cash flow patterns. Its use should be purged from the DOD procedure.

IV. UNDER CAS 414, CONTRACTOR'S PROFIT INCENTIVE WILL BE TOO LOW

Even under certainty and with a perfect capital market, a contractor will not undertake a project for which $NPV(i)=0$. As Jaynes⁵ has put it, "Nobody raises money at $i\%$ in order to reinvest it at $i\%$." To make the project attractive for the contractor, an increment, what Jaynes has called a "profit incentive", must be added to make $NPV(i) > 0$. In government contracting, the usual way in which this is intended to be accomplished is as follows. Let:

$X \equiv$ Total time 0 present value of project costs, all to be reimbursed to the contractor by the government.

$\alpha \equiv$ Desired percentage of X to be paid by the government to the contractor as a profit incentive.

Then, since in a world of certainty there need be no concern with cost overruns or underruns, $X(1+\alpha)$ is the correct time 0 present value of proper government payments to the contractor to cover both his costs and his profit incentive.

But even here, a major problem, which seems to have been inadequately treated in the literature, is that the cost accounting procedures required for use in government contracting may easily lead to an understatement of the true value of X . Then when α is based on such an understated value, the payments made to the contractor may include an actual percentage of the true value, X , which is very much less than what had been intended.

To illustrate what can happen, consider the following simple situation for a proposed contract project. Let:

$I \equiv$ Capital outlay by the contractor to be required at time 0 for equipment. This equipment is to be used for the T period life of the project and, for simplicity, is assumed to have no salvage value.

$W \equiv$ Outlay by the contractor to be required at time 0 for working capital. For simplicity, it is assumed that this remains invested in the project till time T , but is then received back in its entirety, by the contractor.

$H \equiv$ Time 0 present value of all the other costs to be incurred by the contractor for the project.

$d_t \equiv$ Accounting depreciation charge allowed at time t for usage of capital equipment, where $I = d_1 + d_2 + \dots + d_T$.

Then it is straightforward to verify that:

$$X = I + W[1 - (1+i)^{-T}] + H, \text{ and} \quad (4)$$

$$I = \sum_{t=1}^T [d_t + i(I - d_1 - d_2 - \dots - d_{t-1})] (1+i)^{-t}. \quad (5)$$

It follows from (5) that I is equal to the time 0 present value of a series of time t cost components, $d_t + i(I - d_1 - d_2 - \dots - d_{t-1})$, i.e. the time t depreciation charge plus an interest charge based on the book value at the start of the period just ended. But, under Cost Accounting Standard, (CAS), 414, as applied, e.g., in the DOD Guide³ and by Rayburn⁶, the principal upon which interest is figured at time t is $[I - d_1 - d_2 - \dots - d_{t-1} - (d_t/2)]$, i.e. the average book value during the period just ended. These

accounting costs are the ones which are reimbursable to the contractor, but their time 0 present value falls short of the true I by $\sum_{t=1}^T d_t (1+i)^{-t}$, and hence, in using them, X in (4) is understated by that amount. Also, under CAS 414, the cost effect of the contractor's temporary outlay for working capital is completely ignored. Thus, even if it is assumed, for simplicity, that H is properly computed:

$$\begin{aligned} \Delta X &\equiv \text{Amount by which X is understated under rules of CAS 414.} \\ &= \sum_{t=1}^T d_t (1+i)^{-t} + W \left[1 - (1+i)^{-T} \right] \end{aligned} \quad (6)$$

If, e.g., straight-line depreciation is used, $d_t = I/T$ for all t, and (6) reduces to the simpler form:

$$\Delta X = \left[\frac{I}{2T} + W \right] \left[\frac{(1+i)^T - 1}{(1+i)^T} \right] \quad (7)$$

Note that with X understated by ΔX , the time 0 present value of government payments to the contractor, and hence the time 0 present value of his profit incentive, fall short of the correct intended value by $(1+\alpha) \Delta X$. Let:

$$\begin{aligned} \hat{\alpha} &\equiv \text{Actual percentage of X to be paid to the contractor as a profit incentive.} \\ &= \frac{(1+\alpha)(X-\Delta X) - X}{X} = \alpha - (1+\alpha) \frac{\Delta X}{X}. \end{aligned} \quad (8)$$

Since $\Delta X > 0$, inevitably here $\hat{\alpha} < \alpha$. Indeed, under realistic values of the parameters, $\hat{\alpha}$ may even be negative. Suppose, e.g., that $i=0.10$, $T=5$, $I=1$, $W=1$, and $H=0.2$. Then $X=1.579079$, and, with straight-line depreciation, $\Delta X=0.416987$. But then, from (8), even if, say, $\alpha=0.12 > 0$, $\hat{\alpha}=-0.176 < 0$. Such an "incentive" will clearly not encourage the contractor to undertake the project.

Apparently, in practice, compensation is made for such anomalies by ad hoc adjustment of α , the d_t 's, i , and other parameters so as to make the project attractive to the contractor. How much more sense it would make, however, and how much better uniformity and consistency in government contracting would result if the cost accounting standards were consistent with economic theory in the first place.

V. UNCERTAINTY: SHARING FRACTION SHOULD BE PARETO-OPTIMAL

To introduce uncertainty, consider a simple setting in which all cash flows occur at a single point in time, or have been discounted to that time, and in which, once a commitment to undertake the contract or not has been made, all uncertainty is immediately resolved. For a proposed contract

project let:

$b \equiv$ Target fee, i.e. target profit incentive for the contractor to receive.

$x_0 \equiv$ Target cost for the contract to incur.

$x \equiv$ Actual cost the contract incurs, x is assumed to be normally distributed with mean, μ , and variance, σ^2 .

$k \equiv$ Contractor's sharing fraction. If $k=0$, this is a "cost plus fixed fee," (CPFF) contract. If $k=1$, it is "firm fixed price," (FFP). But, more generally, k can also be a fraction.

$F(x) \equiv$ Actual fee the contractor is to receive.

It follows that $F(x) = b + [x_0 - x]k$, where $F(x)$ is normally distributed with mean, $E[F(x)] = b + [x_0 - \mu]k$, and variance, $\sigma^2[F(x)] = \sigma^2 k^2$.

Assume the contractor obeys the standard axioms of rational behavior and thus wishes to maximize his expected utility. Also assume the utility is solely a function of his fee, $F(x)$, and is of simple, risk-averse, exponential form, $U_C[F(x)] = -\exp[-c_1^{-1} F(x)]$, where c_1 is a positive constant, which Pratt⁷ has shown to be an index of risk-aversion, the lower the constant the higher the aversion. Methods for estimating this constant in practice are discussed by Keeney and Raiffa⁸. Hammond⁹ has observed that, under broad circumstances, the exponential is a good approximation to more complex, less tractable utility functions.

Rather than expected utility, it is equivalent and more convenient to work with "risk-adjusted value", (RAV), i.e. the certain amount of money which has the same utility as the expected utility. Here it is straightforward to verify that:

$$\begin{aligned} \text{RAV}_C(k) &\equiv \text{Contractor's risk-adjusted value for the proposed contract.} \\ &= E[F(x)] - \frac{\sigma^2[F(x)]}{2c_1} = b + [x_0 - \mu]k - \frac{\sigma^2}{2c_1} k^2 \end{aligned} \quad (9)$$

Then with all parameters fixed except k , the contractor will choose the k , say \hat{k} , which maximizes $\text{RAV}_C(k)$. In analyses of DOD contracting, it has been common to assume that goal. See, e.g., Scherer.^{1,2}

It has also been common to assume the contractor is risk-neutral, i.e. that he has a linear utility function for money. But if that were true, c_1 would be infinite, and the final term of (9) would disappear. Then, if $[x_0 - \mu] < 0$, $\hat{k} = 0$, and if $[x_0 - \mu] > 0$, $\hat{k} = 1$. The fact that contractors actually choose fractional values of k is thus prima-facie support for the assumption that they are truly risk-averse rather than risk-neutral.

To make \hat{k} higher, and hence get the contractor to assume more contract risk, the government commonly makes target fee, b , increase with k , usually more slowly as k approaches 1. E.g., suppose it increases quadratically; i.e. $b = x_0(\alpha + \beta k - \gamma k^2)$, where α , β , and γ are positive constants. Then, using calculus, it is easy to verify that to maximize $RAV_C(k)$ the contractor will choose:

$$\hat{k} = \frac{(1+\beta)x_0 - \mu}{2\gamma x_0 + 0.5 c_1^{-1} \sigma^2} \quad (10)$$

But, regrettably, it is important to observe that this standard approach of setting $k = \hat{k}$ is not, in general, in the best interest of either the contractor or the government; i.e., it does not yield a Pareto-optimal solution. Without making the government worse off, the contractor can be made better off or vice-versa. To show this, let:

$G(x) \equiv$ Actual fee the government is to pay on the contract, i.e.
actual cost plus actual contractor fee.

Then $G(x) = x + F(x)$, and $G(x)$ is normally distributed with mean, $E[G(x)] = b + \mu + [x_0 - \mu]k$, and variance, $\sigma^2[G(x)] = \sigma^2(1-k)^2$.

Assume the government also obeys the standard axioms of rational behavior and thus wishes to maximize its expected utility. Also assume that utility is solely a function of the fee, $G(x)$, and is of simple, risk-averse, exponential form, $U_G[-G(x)] = -\exp[c_2^{-1} G(x)]$, where c_2 , a positive constant, inversely measures the government's risk-aversion. (Presumably the government is less risk-averse than the contractor, i.e. $c_2 > c_1$, but assumption of that is not essential to the analysis.) Then, with $c \equiv c_1 + c_2$, in the same manner as above it is straightforward to verify the following:

$$\begin{aligned} RAV_G(k) &\equiv \text{Government's risk-adjusted value for the proposed contract.} \\ &= E[-G(x)] - \frac{\sigma^2[-G(x)]}{2c_2} = -b - \mu - [x_0 - \mu]k - \frac{\sigma^2}{2c_2} (1-k)^2 \end{aligned} \quad (11)$$

$$RAV_T(k) \equiv RAV_C(k) + RAV_G(k) = -\mu - \frac{\sigma^2}{2c_2} + \frac{\sigma^2}{c_2} k - \frac{c}{2c_1 c_2} k^2 \quad (12)$$

With all parameters still fixed except k , observe that Pareto-optimality requires choosing the k , say k^* , which maximizes $RAV_T(k)$. Using calculus, it is easy to verify from (12) that:

$$k^* = \frac{c_1}{c_1 + c_2} = \frac{c_1}{c} \quad (13)$$

If $k = k^*$, the only way to make $RAV_C(k^*)$ larger is to adjust b , which, inevitably, makes $RAV_G(k^*)$ smaller, and vice-versa. But if $k \neq k^*$, a switch

to $k=k^*$ would raise $RAV_T(k)$, and, by adjustment of b , the gain could be used to raise either $RAV_C(k)$, or $RAV_G(k)$, or both, without lowering the other. Thus, unless $\hat{k}=k^*$ in (13), \hat{k} in (10) does not yield a Pareto-optimal solution. For a fuller discussion of Pareto-optimality when the sharing parties have exponential utility functions for money, see, e.g., Raiffa¹⁰.

When k^* in (13) is inserted for k in (12), after simplification one finds:

$$RAV_T(k^*) = -\mu - \frac{\sigma^2}{2c} \quad (14)$$

Observe that (14) is exactly the same as the risk-adjusted value of a single party who is to make a payment, x , which is normally distributed with parameters, μ , and σ^2 , and who has a simple, risk-averse, exponential utility function, $U[-x] = -\exp[-cx]$. This is a special case of a useful result, shown, e.g., by Raiffa¹⁰, that whenever two parties have exponential utility functions for money and share outcomes in Pareto-optimal fashion, then for choosing among alternative probabilistic patterns of outcomes, it is optimal for them to work jointly using the exponential group utility function, $U[-x]$, stated above. In the present setting, e.g., using this approach, the merits of various potential pilot plant or other small-scale experiments for reducing the uncertainty might be explored before the contract is undertaken or not and, indeed, even before the experimenting is done.

Then, in general, once the plan with the highest possible $RAV_T(k^*)$ is established, the value of b , which can be zero, positive, or negative, should be set just sufficiently high that $RAV_C(k^*)$ is at the lowest level the contractor will accept. If that leaves $RAV_G(k^*)$ sufficiently high for the government to accept, the contract should be undertaken; otherwise it should not.

VI. CONFLICT: PARETO-OPTIMALITY VS. COST REDUCTION INCENTIVE

Bradley and McCuiston¹¹ claim that the popularity of incentive contracting is not at all because it creates incentive for cost reduction but rather because, by allowing fractional values of k , it permits the contractor to maximize his expected utility or, equivalently, his $RAV_C(k)$. However, contrary to this claim, there is nothing in the theory to imply that incentive for cost reduction, caused by increasing the value of k , might not also be operative. E.g., rather than having expected cost,

μ , being a constant, suppose it decreases linearly with k , reflecting the contractor's greater incentive for cost reduction with higher k ; i.e., $\mu = \mu_0(1-rk)$, where μ_0 is the expected cost when $k=0$, and the constant, $r>0$. Inserting this revised μ into (12), one finds that now:

$$RAV_T(k) = -\mu_0 - \frac{\sigma^2}{2c_2} + \left[r\mu_0 + \frac{\sigma^2}{c_2} \right] k - \frac{c\sigma^2}{2c_1c_2} k^2 \quad (15)$$

Then, again using calculus, it is easy to verify that the value of k , say \hat{k} , which maximizes $RAV_T(k)$ here is:

$$\hat{k} = \left[1 + \frac{\mu_0 r c_2}{\sigma^2} \right] k^* \quad (16)$$

In the special case where $r=0$, (16), of course, reduces to (13), i.e. $\hat{k}=k^*$, but more generally, with $r>0$, (16) also depends on r, μ_0 , and σ^2 . With $r>0$, increased contractor cost reduction effort, induced by increasing k , decreases expected cost, μ , and hence, quite reasonably, $\hat{k} > k^*$.

But note, seemingly paradoxically, this new solution, (16), is not Pareto-optimal. If the contractor is, indeed, able to get the expected cost down to $\mu = \mu_0(1-r\hat{k})$, it would be in his best interest to do that but to accept the lesser value of k , i.e., $k=k^*$. This would raise $RAV_T(k)$, and hence, by adjusting b as appropriate, either he or the government, or both, could be made better off without making the other worse off.

To show this, insert \hat{k} of (16) as k in $RAV_T(k)$ of (15). After simplification, this yields:

$$RAV_T(\hat{k}) = -\mu_0(1-r\hat{k}) - \frac{\sigma^2}{2c} - \frac{c_1c_2(r\mu_0)^2}{2c\sigma^2} \quad (17)$$

Then observe from expression (14) for $RAV_T(k^*)$ under Pareto-optimality, the sum of the first two terms in expression (17) for $RAV_T(\hat{k})$ gives what $RAV_T(k)$ would be if $\mu = \mu_0(1-r\hat{k})$, but $k=k^*$. Thus the third term in (17), reducing it to $RAV_T(\hat{k})$, is a penalty term which is incurred if the higher k , i.e. \hat{k} , must be used to entice the contractor to achieve the lower expected cost, $\mu = \mu_0(1-r\hat{k})$. Knowing this, if the contractor can achieve this expected cost while still having $k=k^*$, b can be raised by the amount of the penalty term, thereby raising $RAV_G(k)$, without lower $RAV_G(k)$. Thus the original solution with $k=\hat{k}$ is not Pareto-optimal. As explained above, and more fully discussed by Raiffa¹⁰, whenever, as here, the two parties have exponential utility functions, Pareto-optimality always requires that $k=k^*$.

VII. RECOMMENDATIONS

The immediate recommendations of this initial study are as follows:

- (1) The DOD's shared savings procedure, based on the internal rate of

return, should be revised at once to be consistent with net present value analysis.

- (2) In economic analysis for incentive contracting, accounting figures based on CAS rules should be revised to reflect actual prospective net cash flows and their timing.
- (3) Models currently recommended for establishing shared savings under uncertainty should be revised to avoid conflict with Pareto optimality.

In addition, substantial further research is recommended as follows. The first part of the initial study considered time streams under certainty, while the rest introduced uncertainty but at a single point in time. What should be explored next is joint consideration of these two approaches. A recent paper by Bernhard¹² introduces this complication using additive exponential utility functions. In the setting of incentive contracting, a comprehensive decision-tree analysis will allow consideration of timing of uncertainty resolution and strategies for sequential decisions, including upper and lower bounds on sharing provisions, constraints on availability of capital, and the relative merits of such things as award fee, termination protection, and renegotiation provisions.

Still further matters to be examined in this follow-on research include complications when utility functions are not risk-averse exponential, e.g. the empirically realistic situation where, for small losses, contractors are risk-seeking rather than risk-averse, implications when contractor and government have different subjective probability distributions, additional discrepancies between accounting and economic concepts, and inclusion of tax effects. Also to be studied are use of alternatives to monetary utility theory including Kahneman and Tversky's¹³ "prospect theory," Bell's¹⁴ suggestion that utility should also be a function of "opportunity loss", and introduction of non-monetary considerations through the multiattribute decision theories of Keeney and Raiffa⁸ and others, e.g. to include such further highly important criteria as meeting of schedule and quality requirements. It is clear from this initial study that theoretical modeling of incentive contracting still offers major opportunity for further useful insights and resultant improved applications.

REFERENCES

1. Scherer, F. M., The Weapons Acquisition Process: Economic Incentives, (Boston: Harvard Graduate School of Business Administration, 1964).
2. Scherer, F. M., "The Theory of Contractual Incentives for Cost Reduction", Quarterly Journal of Economics, Vol. 78, pp. 257-280, (1964).
3. DOD Instruction 5000.XX: DOD Guide for Improving Productivity in Defense Contracting, (Washington, D.C.: Naval Materiel Notice, February 28, 1983).
4. Bernhard, R. H., "A Comprehensive Comparison and Critique of Discounting Indices Proposed for Capital Investment Evaluation," The Engineering Economist, Vol. 16, pp. 157-186, (1971).
5. Jaynes, P. H., Profitability and Economic Choice, (Ames, Iowa: Iowa State University Press, 1968).
6. Rayburn, L. G., Principles of Cost Accounting with Managerial Applications, (Homewood, Illinois: Richard D. Irwin, Inc., 1979).
7. Pratt, J. W., "Risk Aversion in the Small and in the Large," Econometrica, Vol. 32, pp. 122-136, (1964).
8. Keeney, R. L., and H. Raiffa, Decisions with Multiple Objectives: Preferences and Value Tradeoffs, (New York: John Wiley and Sons, Inc., 1976).
9. Hammond, J. S., III, "Simplifying the Choice between Uncertain Prospects Where Preference is Nonlinear," Management Science, Vol. 20, pp. 1047-1072, (1974).
10. Raiffa, H., Decision Analysis: Introductory Lectures on Choices under Uncertainty, (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1968).
11. Bradley, C. E., and C. McCuiston, "The Rationale for Incentive Contracting," Decision Sciences, Vol. 3, pp. 15-30, (1972).
12. Bernhard, R. H., "Risk-Adjusted Values, Timing of Uncertainty Resolution, and the Measurement of Project Worth," Journal of Financial and Quantitative Analysis, Vol. 19, pp. 83-99, (1984).
13. Kahneman, D., and A. Tversky, "Prospect Theory: An Analysis of Decisions under Risk," Econometrica, Vol. 47, pp. 263-291, (1979).
14. Bell, D. E., "Regret in Decision Making under Uncertainty," Operations Research, Vol. 30, pp. 961-981, (1982).

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